24022

B. Tech. (BME) 4th Semester F. Scheme Examination, May–2015

MATHEMATICS–III

Paper–Math–201-F

Time allowed : 3 hours]   [Maximum marks : 100

Note : *Attempt five questions in total selecting one question from each section. Question No. 1 is compulsory.*

1. (a) What are the sufficient conditions for the uniform convergence of a Fourier series.   2½

    (b) Define Fourier Integral of function f(x).   2½

    (c) Prove that $\lim\limits_{z \to 0} \dfrac{\overline{z}}{z}$ does not exist.   2½

    (d) State Cauchy's Integral theorem.   2½

    (e) Define circle of convergence.   2½

    (f) Define discrete and continuous probability distribution.   2½

    (g) Solve the following problem graphically :   5

    Maximize $Z = 2x + 3y$

    subject to the constraints

    $x+y \le 30,\ 3 \le y \le 12,\ x-y \ge 0,\ 0 \le x \le 20$

24022–P-4 Q-9 (15)   [P.T.O.

Section–A

2. (a) Give example of square wave form and hence obtain Fourier series expansion for the same. 10

    (b) Express $f(x) = x$ as a half range series in the interval $0 < x < 2$. 10

3. (a) Using Fourier integral representation, show that

$$\int_0^\infty \frac{\lambda \sin x\lambda}{k^2 + \lambda^2}\, d\lambda = \frac{\pi}{2} e^{-kx}, x > 0, k > 0.$$ 10

    (b) Obtain Fourier transforms of the derivatives

$$\frac{\partial u}{\partial x} \text{ and } \frac{\partial^2 u}{\partial x^2}.$$ 10

Section–B

4. (a) If $a+ib = \tan h\left(v + \frac{i\pi}{4}\right)$, Prove that $a^2+b^2=1$. 10

    (b) State and prove Cauchy-Riemann conditions for an analytic function. 10

5. (a) Show that $\oint_c (z+1)dz = 0$, where C is the boundary of the square whose vertices are at the points $z = 0$, $z = 1$, $z = 1+i$ and $z = i$. 10

(b) Evaluate

$$\oint_c \frac{\sin^6 z}{\left(z - \dfrac{\pi}{6}\right)^3} \, dz,$$

where C is the circle $|z| = 1$. 10

Section–C

6. (a) Find the series expansion of $f(z) = \dfrac{z^2 - 1}{(z+2)(z+3)}$

   in the region (i) $|z| < 2$ (ii) $|z| > 3$. 10

   (b) Evaluate by Cauchy's Residue theorem

   $$\oint_c \frac{z}{(z-1)(z-2)^2} \, dz,$$

   where c is the circle $|z - 2| = \frac{1}{2}$. 10

7. (a) In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B ? 10

   (b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. 10

Section–D

8. (a) Samples of sizes 10 and 14 were taken from two normal populations with S.D. 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. Test whether the means of two populations are the same at 5% level. 10

 (b) Fit a binomial distribution to the following data and test the goodness of fit : 10

x :	0	1	2	3	4	5	6
y :	13	25	52	58	32	16	4

9. Using simplex method

Maximize $Z = x_1 + 2x_2 + x_3$

subject to $2x_1 + x_2 - x_3 \leq 2$, $-2x_1 + x_2 - 5x_3 \geq -6$

$$4x_1 + x_2 + x_3 \leq 6$$

$$x_1 + x_2 + x_3 \geq 0.$$ 20